

Field theory in superfluid ^3He : What are the lessons for particle physics, gravity, and high-temperature superconductivity?

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There are several classes of homogeneous Fermi-systems which are characterized by the topology of the energy spectrum of fermionic quasiparticles: (1) Gapless systems with a Fermi-surface; (2) Systems with a gap in their spectrum; (3) Gapless systems with topologically stable point nodes (Fermi points); and (4) Gapless systems with topologically unstable lines of nodes (Fermi lines). Superfluid ^3He -A and electroweak vacuum belong to the universality Class (3). The fermionic quasiparticles (particles) in this class are chiral: they are left-handed or right-handed. The collective bosonic modes of systems of Class (3) are the effective gauge and gravitational fields. The great advantage of superfluid ^3He -A is that we can perform experiments using this condensed matter and thereby simulate many phenomena in high energy physics, including axial anomaly, baryoproduction, and magnetogenesis. ^3He -A textures induce a nontrivial effective metrics of the space, where the free quasiparticles move along geodesics. With ^3He -A one can simulate event horizons, Hawking radiation, rotating vacuum, etc. High-temperature superconductors are believed to belong to Class (4). They have gapless fermionic quasiparticles with a "relativistic" spectrum close to gap nodes, which allows application of ideas developed for superfluid ^3He -A.

I. INTRODUCTION

It is now well understood that the Universe and its symmetry-broken ground state – the physical vacuum – may behave like a condensed matter system with a complicated and possibly degenerate ground state [1–4].

If the analogy of the quantum mechanical physical vacuum with condensed matter systems is taken seriously, the first question which arises is: Which system of condensed matter reproduces most closely the properties of the quantum vacuum? Since particle physics deals with interacting Fermi and Bose quantum fields, the system should be fermionic. This requirement excludes superfluid ^4He , which contains only Bose fields. In Fermi systems, such as metals, superconductors, and normal and superfluid ^3He , in addition to the fermionic degrees of freedom which come from the bare fermions, electrons, and ^3He -atoms, respectively, the quantum Bose fields appear as low-energy collective modes. Therefore, these systems do represent interacting Fermi and Bose quantum fields.

Which Fermi system is the best? To answer this question we must first realize that the main feature which differentiates between various Fermi systems is the topology of the quasiparticle spectrum in the low energy (infrared) corner. We will consider only systems whose ground state is spatially homogeneous – this is one of the least disputed property of the physical vacuum. When the topology of the quasiparticle spectrum is taken into ac-

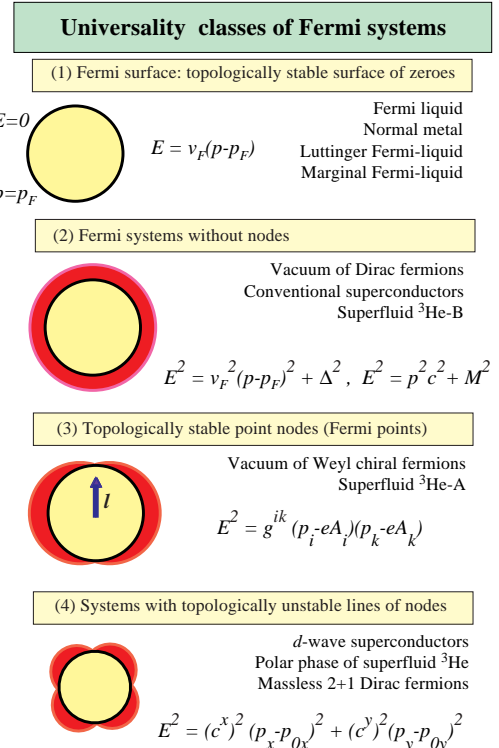


FIG. 1. Universality classes of the fermionic ground state (vacuum).

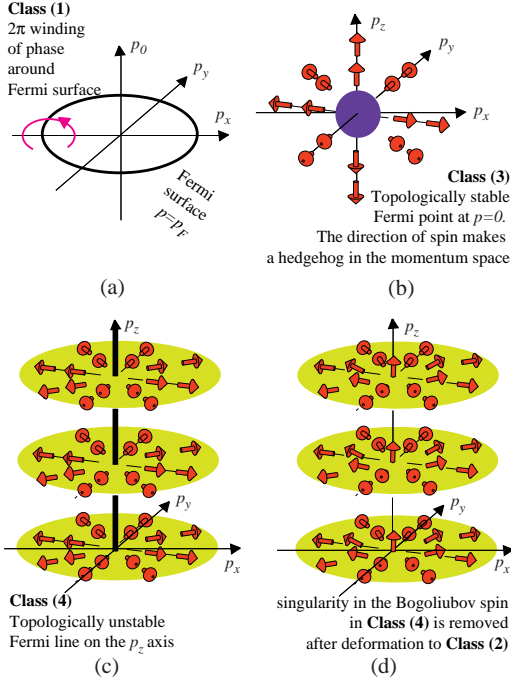


FIG. 2. (a) Winding of the propagator phase around the Fermi surface. For simplicity the p_z coordinate is hidden so that the Fermi surface is the line $(p_0 = 0, p = p_F)$ in the 2+1 momentum space. This line is a singularity, which is similar to a vortex in a real 3D-space: The phase of the propagator $G = (ip_0 - (p_x^2 + p_y^2 - p_F^2)/2m)^{-1}$ changes by 2π around the line in the momentum space in the same manner as the phase of the order parameter changes by 2π around a vortex in the real space. (b) Fermi point at $\mathbf{p} = 0$ in the 3D momentum space (p_x, p_y, p_z) . At this point the particle energy $E = cp$ is zero. A right-handed particle is considered with its spin parallel to the momentum \mathbf{p} , *i.e.* $\mathbf{s}(\mathbf{p}) = (1/2)\mathbf{p}/p$. The spin makes a hedgehog in the momentum space, which is topologically stable. (c) Fermi line – topologically unstable manifold of zeroes – is shown in the 3D momentum space (p_x, p_y, p_z) . The (Bogoliubov) spin (arrows) is confined into the (p_x, p_y) plane and has a singularity on the p_z axis. (d) This singularity can be removed by a continuous transformation. The spin escapes into a third dimension (p_z) and becomes well defined on the p_z axis. As a result, the quasiparticle spectrum becomes fully gapped (the “relativistic” fermion acquires the mass).

count, the homogeneous Fermi systems can be organized into very few classes (see Fig. 1).

II. SYSTEMS WITH A FERMI SURFACE

The most common universality class is made of fermionic systems which have a Fermi surface (FS). Any collection of weakly interacting fermionic particles belongs to this class. In the extreme limit of a noninteracting Fermi gas, with an energy spectrum $E(p) = p^2/2m - \mu$, where μ is the chemical potential, the FS

bounds the volume in the momentum space where $E(p) < 0$ and where the quasiparticle states are all occupied at $T = 0$. In this isotropic model the FS is a sphere of radius $p_F = \sqrt{2m\mu}$. It is remarkable that the FS survives even if interactions between particles are introduced. This stability is a topological property of the FS which is reflected in the Feynman quantum mechanical propagator $G = (z - \mathcal{H})^{-1}$ for the particle (the one-particle Green’s function).

Let us write the propagator for a given momentum \mathbf{p} and for the imaginary frequency, $z = ip_0$ (The imaginary frequency is introduced to avoid the conventional singularity of the propagator at $z = E(p)$). For noninteracting particles the propagator has the form $G = (ip_0 - E(p))^{-1}$. Obviously there is still a singularity: On the hypersurface $(p_0 = 0, p = p_F)$ in the 4-dimensional space (p_0, \mathbf{p}) the propagator is not well defined. What is important is that this singularity is stable: The phase Φ of the Green’s function $G = |G|e^{i\Phi}$ changes by 2π around the path embracing this surface in the 4D-space (see Fig. 2), and the phase winding number is robust towards any perturbation. Thus the singularity of the Green’s function on the 2D-surface in the momentum space is preserved, even when interactions between particles are introduced.

Exactly the same topological conservation of the winding number leads to the stability of the quantized vortex in superfluids and superconductors, the only difference being that, in the case of vortices, the phase winding occurs in the real space (see Fig. 3), instead of the momentum space. The connection between the topology in real space and the topology in momentum space is, in fact, even deeper (see *e.g.* Refs. [5,6]).

The topology of the propagator in the 4D-momentum space is thus essential for the Landau theory of an interacting Fermi liquid; it confirms the assumption that in Fermi liquids the spectrum of quasiparticles at low energy is similar to that of particles in a Fermi gas. The interactions do not change the topology of the fermionic spectrum but they produce the effective field acting on a given particle by the other moving particles. While this effective field cannot destroy the FS owing to its topological stability, it can shift its position locally. Therefore, a collective motion of the particles is seen by an individual quasiparticle as a dynamical mode of the FS. These bosonic oscillative modes are known as different harmonics of the zero sound. An example is shown in the upper part of Fig. 4.

III. SYSTEMS WITH A FERMI POINT

Although the systems we have discussed contain fermionic and bosonic quantum fields, this is not the relativistic quantum field theory we need: There is no Lorentz invariance and the oscillations of the Fermi surface do not resemble the gauge field even remotely. The

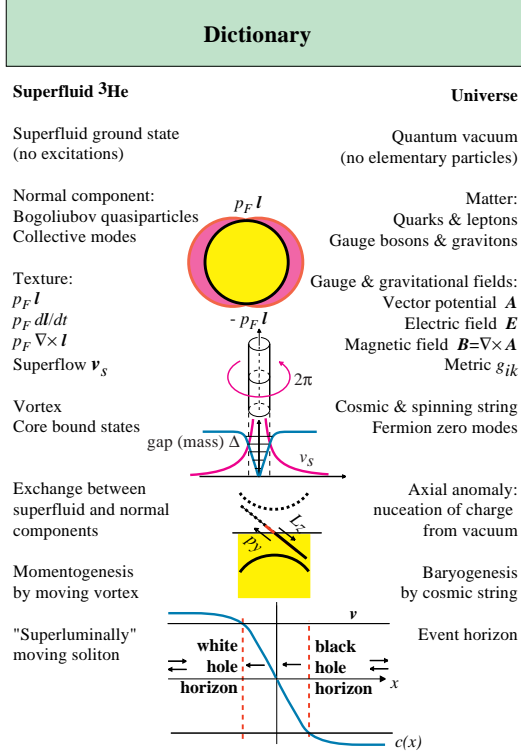


FIG. 3. Dictionary

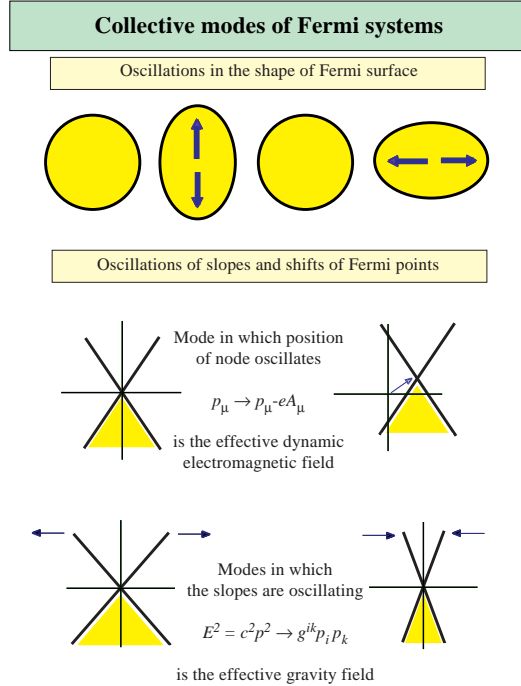


FIG. 4. Collective modes of fermionic systems

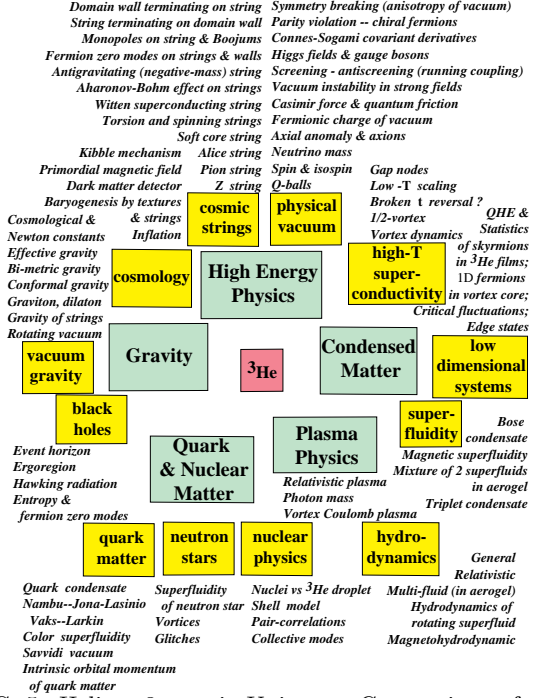


FIG. 5. Helium-3-centric Universe: Connections of superfluid ^3He to other branches of physics. For each item one can find the analogous phenomenon in superfluid phases of ^3He . The analogies can result from the symmetry, from topology in real space or from topology in momentum space. In many cases they are described by the same equations.

situation is somewhat better for Class (2), *i.e.* for fermionic systems with fully gapped spectra; examples, which provide useful analogies with Dirac fermions and spontaneously broken symmetry in quantum fields, are conventional superconductors [7] and superfluid $^3\text{He-B}$ [8]. The latter also serves as a model system for simulations of many phenomena in particle physics and cosmology (see Fig. 5), including experimental verification [9] of the Kibble mechanism describing formation of cosmic strings in the early Universe [10].

However, we proceed to Class (3) which most fully exhibits the fundamental properties needed for a realization of the relativistic quantum fields, analogous to those in particle physics and gravity.

Class (3) systems, whose representatives are superfluid $^3\text{He-A}$ and the vacuum of relativistic left-handed and right-handed chiral fermions, is characterized by points in the momentum space where the (quasi)particle energy is zero. In particle physics the energy spectrum $E(\mathbf{p}) = cp$ is characteristic of the massless neutrino (or any other chiral lepton or quark in the Standard Model of electroweak interactions) with c being the speed of light. The energy of a neutrino is zero at point $\mathbf{p} = 0$ in the 3D momentum space. In condensed matter systems such point nodes have been realized first in superfluid $^3\text{He-A}$, which

we discuss later. The Hamiltonian for the neutrino – the massless spin-1/2 particle – is a 2×2 matrix $\mathcal{H} = \pm c\vec{\sigma} \cdot \mathbf{p}$, which is expressed in terms of the Pauli spin matrices $\vec{\sigma}$. The sign $+$ is for a right-handed particle and $-$ for a left-handed one: the spin of the particle is oriented along or opposite to its momentum, respectively.

Let us again consider the propagator of the particle $G = (ip_0 - \mathcal{H})^{-1}$ on the imaginary frequency axis, $z = ip_0$. One can see that this propagator still has a singularity, which is now not on the surface but at point $(p_0 = 0, \mathbf{p} = 0)$ in the 4D momentum space. It is important that this Fermi point is not simply the shrunk Fermi surface which is topologically unstable and can disappear. The discussed points control the topological stability; they cannot be destroyed by external perturbations of the system.

Such stability can be visualized if one considers the behavior of the particle spin $\mathbf{s}(\mathbf{p})$ as a function of its momentum \mathbf{p} in the 3D-space $\mathbf{p} = (p_x, p_y, p_z)$. For right-handed particles $\mathbf{s}(\mathbf{p}) = \mathbf{p}/2p$, while for left-handed ones $\mathbf{s}(\mathbf{p}) = -\mathbf{p}/2p$. In both cases the spin distribution in the momentum space looks like a hedgehog (see Fig. 2b), whose spines are represented by spins: spines point outward for the right-handed particle and inward for the left-handed one. In the 3D-space the hedgehog is topologically stable: There exists an integer topological invariant, which supports the stability of the Fermi point in the same manner as the conservation of the winding number is responsible for the stability of a vortex line and the Fermi surface. This invariant can be expressed in terms of the propagator [11].

The consequence is the following: The effective fields acting on a given particle due to interactions with other moving particles cannot destroy the Fermi point. They lead to a shift in its position in the momentum space and to a change of the slopes of the energy spectrum (see Fig. 4). This means that the low-frequency collective modes in such Fermi liquids are the propagating collective oscillations of the positions of the Fermi point and of the slopes at the Fermi point. The former is felt by the right- or the left-handed quasiparticles as the gauge (electromagnetic) field, because the main effect of the electromagnetic field $A_\mu = (A_0, \mathbf{A})$ is just a dynamical change in the position of zero in the energy spectrum: $(E - eA_0)^2 = c^2(\mathbf{p} - e\mathbf{A})^2$.

The latter, *i.e.* the change of the slope, corresponds to a change in the speed of light, which can be different for different directions in space: $c^x = c + \delta c^x$, $c^y = c + \delta c^y$, $c^z = c + \delta c^z$. In a more general consideration the energy spectrum in the perturbed state is expressed in terms of the matrix of slopes, $E^2 = g^{ik}p_ip_k$. In the physical sense this matrix is the metric tensor: The quasiparticles feel the inverse tensor g_{ik} as the metric of the effective space in which they move along the geodesic curves with the interval $ds^2 = -dt^2 + g_{ik}dx^idx^k$. Therefore, the collective modes related to the slopes play the part of the gravity

field (see Fig. 4).

The most general form of the energy spectrum close to the Fermi point, *i.e.* at low energies, is $g^{\mu\nu}(p_\mu - eA_\mu)(p_\nu - eA_\nu) = 0$, which describes a relativistic massless (actually chiral) particle moving in the electromagnetic and gravity fields. It is most important that this is the general form of the energy spectrum in the vicinity of any Fermi point, even if the underlying Fermi system is not Lorentz invariant; superfluid $^3\text{He-A}$ is an example. The fermionic spectrum necessarily becomes Lorentz invariant near the Fermi point, *i.e.* this invariance is not fundamental but a low-energy property of any system with a Fermi point.

Another important property which results from the above equation is that the fermionic propagator near the Fermi point is gauge invariant and even invariant under general coordinate transformations (general covariance). For example, the local phase transformation of the wave function of the fermion, $\Psi \rightarrow \Psi e^{ie\alpha(\mathbf{r},t)}$ can be compensated by a shift of the "electromagnetic" field $A_\mu \rightarrow A_\mu + \partial_\mu\alpha$. Such invariances are usually attributed to fundamental properties of electromagnetic (A_μ) and gravitational ($g^{\mu\nu}$) fields, but here they arise spontaneously as low-energy phenomena.

What about equations for these collective bosonic modes, A_μ and $g^{\mu\nu}$? Are they also gauge invariant, *i.e.* invariant under transformation $A_\mu \rightarrow A_\mu + \partial_\mu\alpha$? Also, do they obey the general covariance? In other words, do they correspond to Maxwell and Einstein equations for electromagnetic and gravitational fields, respectively? The answer to this question depends on the structure of the underlying Fermi system. Let us discuss this.

The effective Lagrangian for the collective modes is obtained by integrating over the vacuum fluctuations of the fermionic field in the presence of the collective bosonic fields. This principle was used by Sakharov and Zeldovich to obtain an effective gravity [12] and an effective electrodynamics [13], both arising from fluctuations of the fermionic vacuum.

Let us suppose that the main contribution to the effective action comes from the vacuum fermions whose momenta \mathbf{p} are concentrated near the Fermi point. Since these "relativistic" fermions, moving in "gauge" and "gravity" fields, obey gauge invariance and general covariance, the result of the integration – the effective Lagrangian for the bosonic fields – is also Lorentz invariant, gauge invariant, and even has a covariant form. In this case the obtained effective Lagrangian does give Maxwell equations for A_μ [13] and Einstein equations for $g_{\mu\nu}$ [12], so that the propagating bosonic collective modes do represent the gauge bosons and gravitons.

In the extreme limit, when the massless relativistic fermions are overdominating in vacuum, a new invariance is revealed – the conformal invariance: The conformal transformation $g_{\mu\nu} \rightarrow ag_{\mu\nu}$ leaves the massless fermions intact, as a result the effective action for gravity becomes

the conformally invariant Weyl action. Weyl gravity is a viable rival to Einstein gravity in modern cosmology [14,15].

Thus, if two requirements are fulfilled – (i) the fermionic system has a Fermi point and (ii) the main physics is concentrated near this Fermi point – the system acquires at low energy all the properties of modern quantum field theory: chiral fermions, quantum gauge fields, and gravity. All these ingredients are actually low-energy (infra-red) phenomena.

As a practical realization of Class (3) Fermi systems in condensed matter, let us consider excitations in $^3\text{He-A}$ (see Fig. 3). After the transition to the superfluid state the Fermi surface disappears, a gap appears instead in the quasiparticle energy spectrum. Distinct from conventional superconductors belonging to Class (2), the gap has nodes at the north and south poles of the former Fermi surface (at $\mathbf{p} = \pm p_F \hat{\mathbf{l}}$, where $\hat{\mathbf{l}}$ is the direction of spontaneous angular momentum in $^3\text{He-A}$). Each node is a topologically stable Fermi point with left-handed quasiparticles near the north pole and right-handed quasiparticles near the south pole. Another example of the Fermi point in condensed matter has been discussed for gapless semiconductors [16].

Close to the gap nodes, *i.e.* at energies $E \ll \Delta_0$, where Δ_0 is the maximal value of the gap in $^3\text{He-A}$, playing the part of the Planck energy, the quasiparticles obey the relativistic equation $g^{\mu\nu}(p_\mu - eA_\mu)(p_\nu - eA_\nu) = 0$. Here $e = \pm$ is the "electric charge" and simultaneously the chirality of the quasiparticles. The effective electromagnetic field is induced by the dynamical $\hat{\mathbf{l}}$ -field and by the velocity \mathbf{v}_s of the superfluid quantum vacuum of $^3\text{He-A}$: $\mathbf{A} = p_F \hat{\mathbf{l}}$, $A_0 = p_F \mathbf{v}_s \cdot \hat{\mathbf{l}}$. This means, *e.g.* that the texture of the $\hat{\mathbf{l}}$ -vector is felt by quasiparticles as the magnetic field according to equation $\mathbf{B} = p_F \vec{\nabla} \times \hat{\mathbf{l}}$ (see Fig. 3). Moreover, in the low-energy limit, the A_μ field does obey Maxwell equations: The integration over the vacuum fermions is concentrated near the Fermi point due to logarithmic divergence, known in particle physics as the zero charge effect.

The metric of the effective space, where the chiral quasiparticles move along the geodesic curves, has the components: $g^{ik} = c_\perp^2 (\delta^{ik} - \hat{l}^i \hat{l}^k) + c_\parallel^2 \hat{l}^i \hat{l}^k - v_s^i v_s^k$, $g^{00} = -1$, $g^{0i} = v_s^i$. The quantities c_\parallel and c_\perp – the velocities of "light" propagating along and transverse to $\hat{\mathbf{l}}$ – are expressed by the "fundamental" parameters of $^3\text{He-A}$: $c_\parallel = p_F/m^*$, $c_\perp = \Delta_0/p_F$, where m^* and p_F are the mass and Fermi momentum, respectively, of quasiparticles in the normal ^3He . Since $\Delta_0 \ll E_F \sim p_F^2/2m^*$, the "Einstein" and "Weyl" actions for $g_{\mu\nu}$ are highly contaminated by many noncovariant terms, which come from the integration over the "nonrelativistic" high energy degrees of freedom in the region $\Delta_0 < E < E_F$.

In spite of the absence of general covariance in general, many different properties of the physical vacuum

with a Fermi point, whose direct observation is still far from realization, can be simulated in $^3\text{He-A}$. One of them is the chiral anomaly, which allows the nucleation of the fermionic charge from the vacuum [17,18] (see Figs. 3, 5). Since the chiral anomaly is a low-energy phenomenon, the anomaly equation in $^3\text{He-A}$ has gauge invariant and general covariant form, and thus exactly coincides with that derived by Adler and by Bell and Jackiw [17,18]. This equation has been verified in several ^3He experiments [19,20]. In particle physics, the only evidence of axial anomaly is related to the decay of the neutral pion $\pi^0 \rightarrow 2\gamma$, although the anomaly is much used in different cosmological scenarios explaining an excess of matter over antimatter in the Universe (see review [23]).

The advantage of $^3\text{He-A}$ for such simulations is that the theory of this superfluid is in some sense complete: At least in principle one can derive everything from the bare ^3He atoms interacting via a known potential. This is why there is no cut-off problem: we know (or can calculate from first principles) what happens not only in the low energy limit, where the fermionic spectrum is gauge invariant or covariant, but also at higher energy, where the Lorentz and gauge invariances are violated. This allows us to investigate problems which require knowledge of physics beyond the Planck cut-off, *e.g.* the quantum effects related to the event horizon of black holes. It is also important that there is a variety of textures in superfluid $^3\text{He-A}$ which allow us to simulate the event horizon and ergoregion when the texture moves with velocity exceeding the local speed of "light" [21] or rotates [22] (see Figs. 3, 5).

IV. SYSTEMS WITH A FERMILINE

The high-temperature superconductors in cuprates most probably contain zeroes in their quasiparticle energy spectrum. The ARPES experiments [24] show that these are lines in the 3D momentum space where the quasiparticle energy is zero or, equivalently, point zeroes in the 2D CuO_2 planes. The high-T superconductors thus belong to Class (4) of systems with Fermi lines: the dimension D of the manifold of zeroes is 1, which is intermediate between a Fermi surface with $D = 2$ and a Fermi point with $D = 0$.

As in the other two classes, all low-energy (or infra-red) properties of cuprate superconductors are determined by zeroes. In particular, the density of the fermionic states with energy E is determined by the dimension of the zeroes: $N(E) = \sum_{\mathbf{p}} \delta(E - E(\mathbf{p})) \sim E^{2-D}$. Many low-temperature properties of these superconductors are obtained from a simple scaling, known in $^3\text{He-A}$ (this represents one of the connections in Fig. 5). For example, an external magnetic field B has the dimension of E^2 . At finite B , the density of states is nonzero even at $E = 0$ and equals $N(0, B) \sim B^{(2-D)/2}$ [25]. An experimental

indication of such scaling with $D = 1$ was reported for $\text{YBa}_2\text{Cu}_3\text{O}_7$ in Ref. [26].

The energy spectrum of quasiparticles near each of the 4 gap nodes in Fig. 1 can be written as $E = \tau_1 c^x(p_x - eA_x) + \tau_3 c^y(p_y - eA_y)$, where $\vec{\tau}$'s are the Pauli matrices in the Bogoliubov-Nambu particle-hole space. The "speeds" of light c^x and c^y are the "fundamental" characteristics determined by the microscopic physics of cuprates; \mathbf{A} is the effective (not electromagnetic) field, which indicates the position of the nodal lines in the momentum space. This means that the system belongs to the same class as the 2+1 quantum electrodynamics with massless fermions.

The lines of zeroes generally have no topological stability: The singular line in the momentum space from which the spines (now the vector $\vec{\tau}$) point outward (see Fig. 2c) can be eliminated by the escape of the $\vec{\tau}$ -vector to a third dimension. This may be accomplished by an operation similar to the folding of an umbrella (see Fig. 2d).

Existence of the nodal lines can be prescribed, however, by the symmetry of the ground state. There are many nontrivial classes of superconductors, whose symmetry supports the existence of nodal lines in symmetric positions in the momentum space [27]. The symmetry violating perturbations, such as impurities, an external magnetic field, *etc.*, destroy the lines of zeroes [6]. One could expect different types of transformations of these lines of zeroes which depend on the perturbation. Impurities, for example, can: (i) produce the gap in the fermionic spectrum [28], thus transforming the system to Class (2) (see Fig. 2d); (ii) lead to a finite density of states [29], thus transforming the system to Class (1); (iii) produce zeroes of fractional dimension, which means that the exponent in the density of states $N(E) \propto E^{2-D}$ is non-integral [30] and thus corresponds to a fractional dimension D of the manifold of zeroes; and (iv) lead to localization [31]. An open question is: Can the quantum fluctuations do the same, in particular, can they change the effective dimension of the zeroes?

V. CONCLUSION

The fermionic systems with topologically stable Fermi points have a remarkable property: In the low-energy corner the system exhibits an enhanced symmetry. The Lorentz invariance, general covariance, gauge invariance, and conformal invariance all appear spontaneously in this corner and bring with them chiral relativistic fermions, gauge fields, and gravity. All are low-energy phenomena, which are absent at higher energies. In particular, this suggests that gravity exists only in the infrared limit, *i.e.* only low-energy gravitons can be quantized [32].

Distinct from the string theory, which also gives rise to gravitation, the Fermi point mechanism does not require

a high dimensionality for the space-time: The topologically stable Fermi point is a property of the conventional 3+1 dimensional space-time.

There are actually two main guesses about the symmetry at high energies: (i) Conventional wisdom prescribes a higher symmetry at higher energies: $SO(10)$, supersymmetry, *etc.*; (ii) A contrary conjecture is that all symmetries known in the Universe disappear at higher energies when the Planck energy is approached. This includes the Lorentz invariance [33], whose violation at high energies can be the origin of the observed neutrino oscillations [34]. The condensed matter analogy with Fermi points supports the second guess.

At "very low" energies of the electroweak scale $E_{\text{ew}} \sim 100$ GeV, the chiral fermions acquire masses and become the Dirac fermions of Class (2). There are also two main guesses how this happens and both can be described in terms of the Fermi points: (i) The Standard Model of symmetry breaking. From the point of view of momentum-space topology, it corresponds to the coalescence and mutual annihilation of Fermi points with opposite topological invariants. (ii) In the alternative theory, the mass matrix of fermions appears in the same way as the gauge field (see *e.g.* [35,36]). Using the language of the Fermi points, the gauge fields, the Higgs fields, and Yukawa interactions, all are realized as shifts of positions of Fermi points corresponding to different quarks and leptons. It is interesting that this way of unification of gauge and Higgs fields, which has been called "generalized covariant derivative", is known in ${}^3\text{He-A}$, where most of effective gauge fields come from the collective modes of the order parameter, *i.e.* from the Higgs field.

However, at the moment it is not clear whether the Fermi point is really the true way of how all the low-energy phenomena arise in the physical vacuum. Before making a conclusion one should manage to construct a scenario of how the 45 chiral fermions of 3 generations (or 48 fermions, if neutrinos have a mass) and 12 gauge bosons of the $SU(3) \times SU(2) \times U(1)$ group of strong and electroweak interactions can arise as effective Fermi and Bose fields from the Fermi points.

${}^3\text{He-A}$ gives some hints that this may be possible. The Fermi point (say, at the north pole) is actually doubly degenerate owing to the ordinary spin of the ${}^3\text{He}$ atom. The double degeneracy results in the $SU(2)$ effective gauge field, acting on quasiparticles near the Fermi point [37,11]. This means that the higher symmetry groups could be a consequence of the Fermi point degeneracy. For example, the 4-fold degeneracy could result in the 4 left-handed + 4 right-handed fermionic species and simultaneously in the $SU(4)$ gauge group.

In this example, however, the number of bosons exceeds the number of fermions. To obtain the correct number of bosons and fermions would probably require a composite model for quarks and leptons. There is, how-

ever, another possibility how to reduce the number of the effective gauge fields. Chadha and Nielsen [33] considered the massless electrodynamics with different metrics for the left-handed and right-handed fermions; their model thus violates the Lorentz invariance. They found that the two metrics converge to a single one as the energy is lowered. Thus in the low-energy corner the Lorentz invariance becomes better and better, and at the same time the number of independent bosonic modes decreases.

The violation of all invariances at high energy imposes another problem to be solved: Why are the corrections due to noninvariance extremely small at low energies? Actually none of the corrections have been experimentally identified so far. This means that the integration over the fermionic vacuum, which produces the action for the gauge and gravity fields, is very effectively concentrated near the Fermi points, where all the symmetries are present. There should be a special mechanism, such as an enhanced quasiparticle relaxation at higher energy, which effectively switches off the nonsymmetric high-energy contribution. The same mechanism could be responsible for the absence of the cosmological term in the Einstein equations.

As we have already mentioned, with given physical parameters ${}^3\text{He-A}$ is not a good model for such effective cancellation: While the Maxwell action is really dominating at low energies due to the logarithmically running coupling constant, the Einstein action is polluted by the noncovariant terms, since the contribution of the vacuum fermions far from the Fermi point becomes dominating. To remove the polluting terms, the integration must be spontaneously cut-off at energies much below the "Planck" scale, $E \ll \Delta_0$. This produces strong limitations on the parameters of the underlying condensed matter.

Nevertheless many phenomena related to the physical vacuum have been or could be visualized in ${}^3\text{He-A}$. There are many other connections between superfluid ${}^3\text{He}$ and the rest of physics which should be exploited as well (see Fig. 5).

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